Complexity of cycle-transverse matching problems

IWOCA 2011

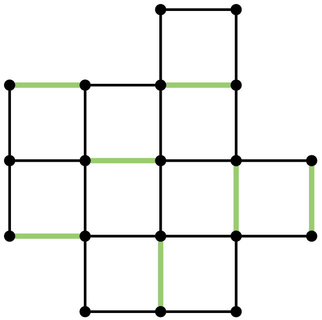
Ross Churchley

University of Victoria

June 18, 2011

Definition

A C_k-**transverse matching** M is one which breaks every cycle of length k



A C_4-transverse matching.

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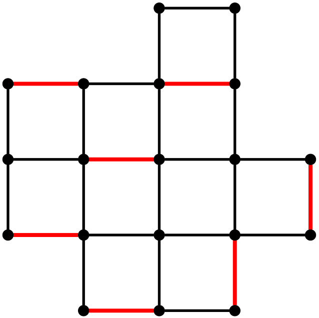

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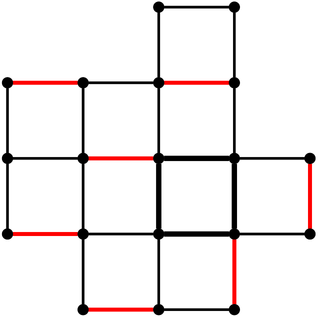

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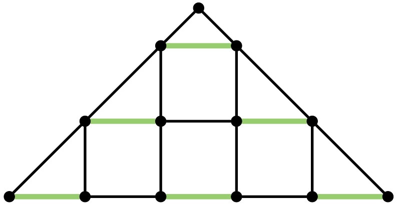

C_3- and C_4-transverse, but not C_5-transverse

tie-in with previous talk

Tatami tilings

Image from flickr user Joel Abroad

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Image from Le Blog du Judo Club de St James

tatami mats meet in T, not +

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"auspicious arrangements" date from Edo period (1603-1868)

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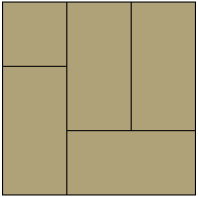

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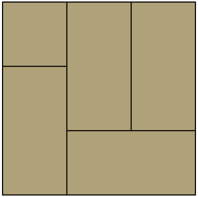

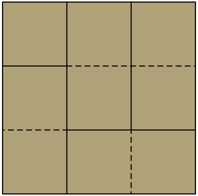

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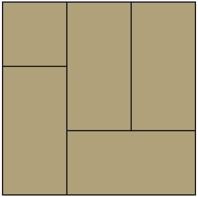

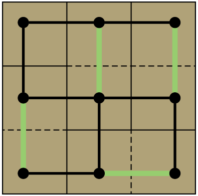

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complexity of finding C_k-transverse matchings

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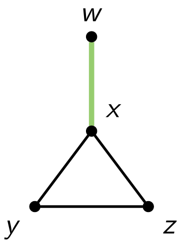
$$k = 3$$

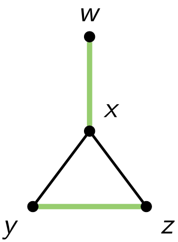
Finding C_3-transverse matchings

If M is a C_3-transverse matching containing wx



Finding C_3-transverse matchings

If M is a C_3-transverse matching containing wx then it also contains yz.



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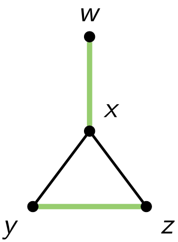

Definition / horrible pun

If M has this property, it is a **pawssible matching**.

Proposition

Let M, M_e be pawssible matchings such that $e \in M_e$ is minimal. If $M \cup e$ is a matching, then $M \cup M_e$ is a pawssible matching.

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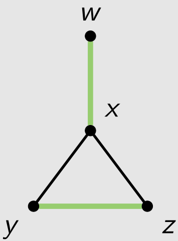

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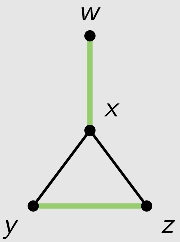

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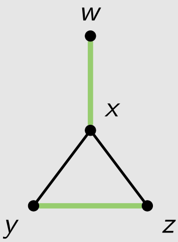

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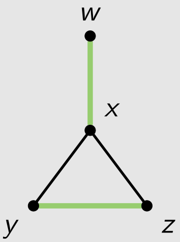

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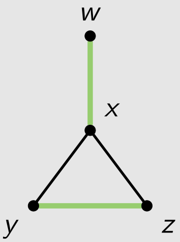

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Corollary

If M is a pawssible matching, then it can be extended to a C_3-transverse matching (provided G admits one).

Theorem

It can be determined in polynomial time whether a graph G admits a C_3-transverse matching.

complexity of finding C_k-transverse matchings

$$k \geq 4$$

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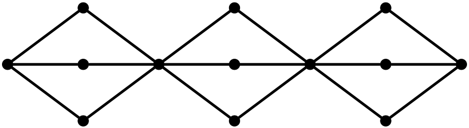

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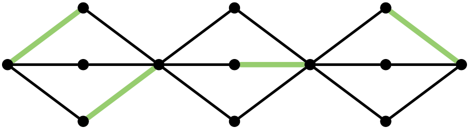

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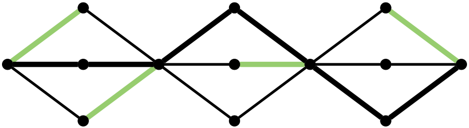

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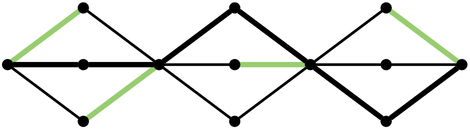
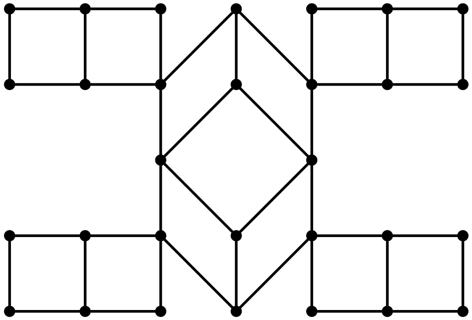


Build a cycle out of a path P and an "accordion path." Then any C_k-transverse matching contains at least one edge from P.

C_4-transverse matchings

Unfortunately, accordion paths don't work for $k = 4$. Reduction is from 4-SAT with ad-hoc gadgets:



graph partition problems

Problem: partition the vertices of a graph into A, B inducing F_A-free and F_B-free subgraphs, respectively.

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F_A	F_B	Chordal	Perfect	Line	Claw-free	All graphs
K_2	P_3	P	NPC	P	P	NPC

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K_2	C_{k+4}					

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K_2	C_3			P	?	NPC
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K_2	K_{k+4}	P	P	P	?	NPC
K_2	co-P_3	P	P	P	P	P
K_2	co-C_3	P	P	P	P	P

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K_2	co-P_3	P	P	P	P	P
K_2	co-C_3	P	P	P	P	P
K_2	$2C_3$	?	?	?	?	?

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