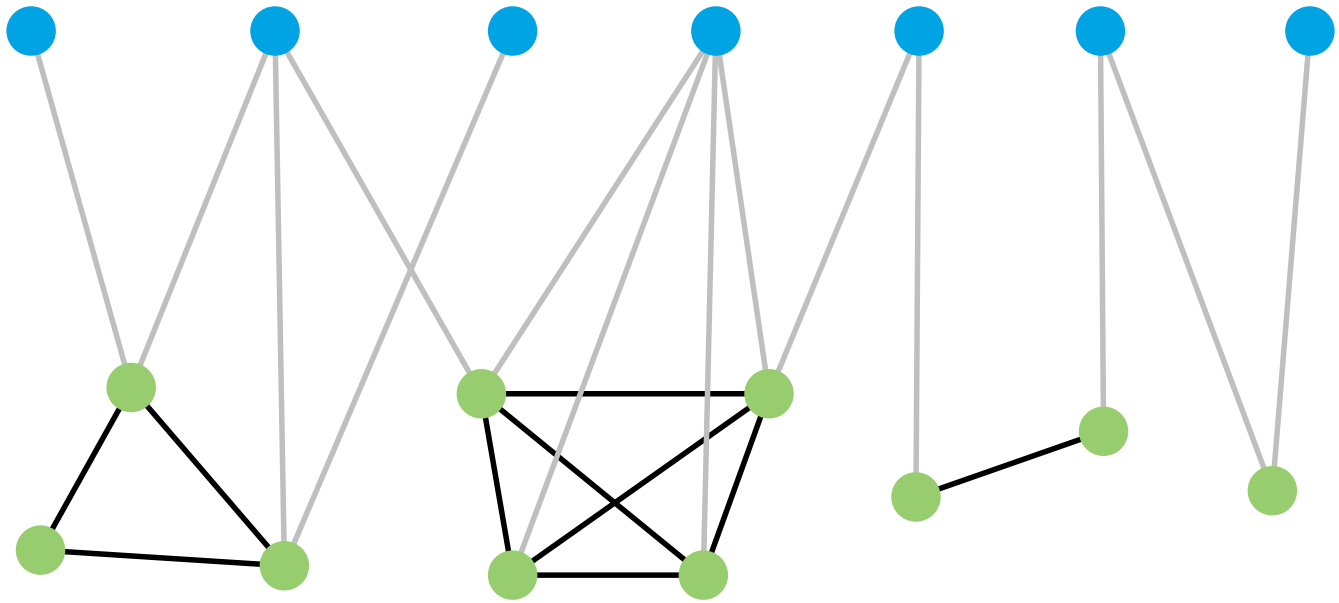
# Monopolar claw-free graphs

Ross Churchley

University of Victoria

December 4, 2010

# A monopolar partition



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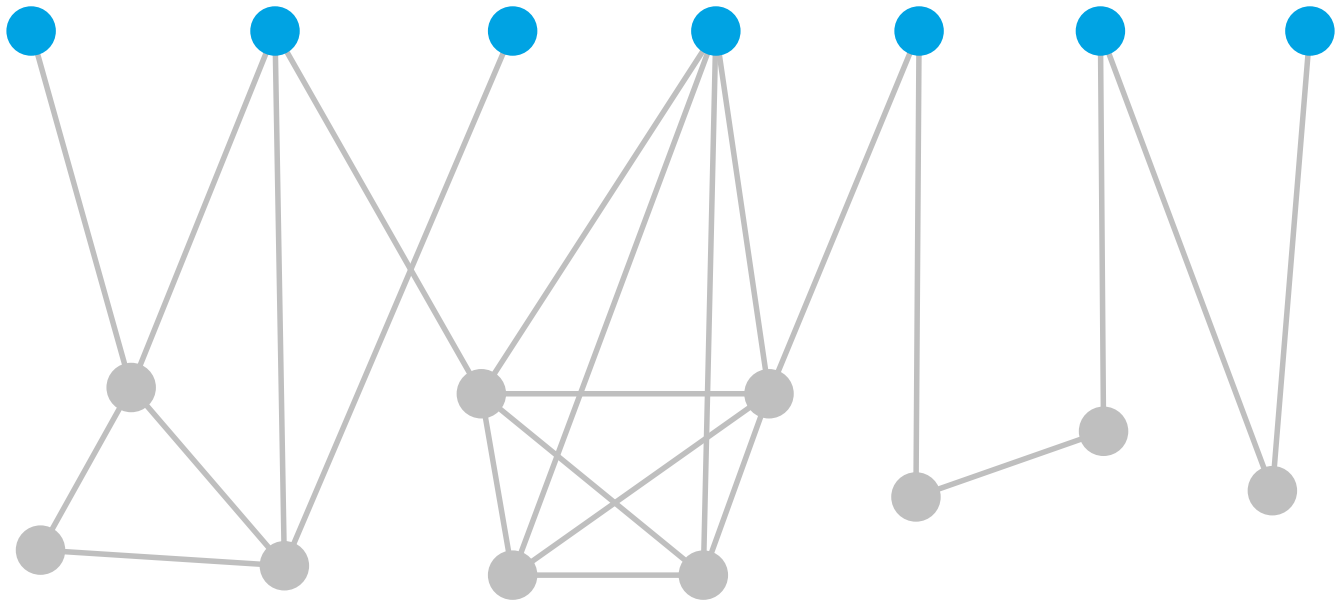

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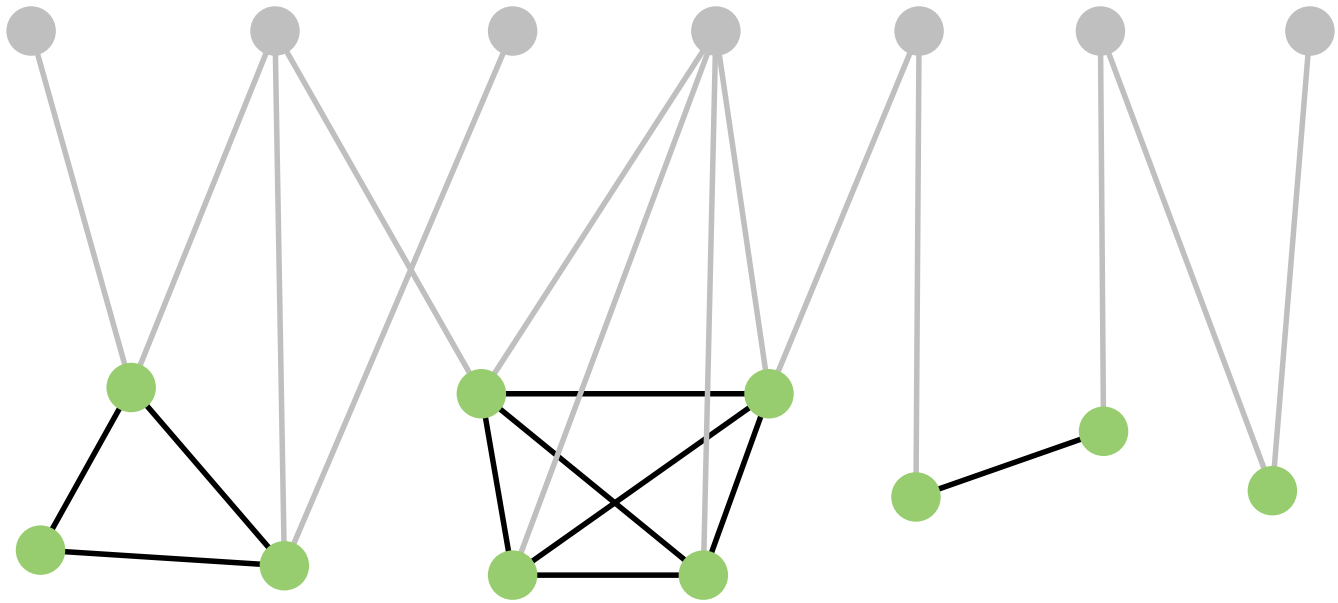
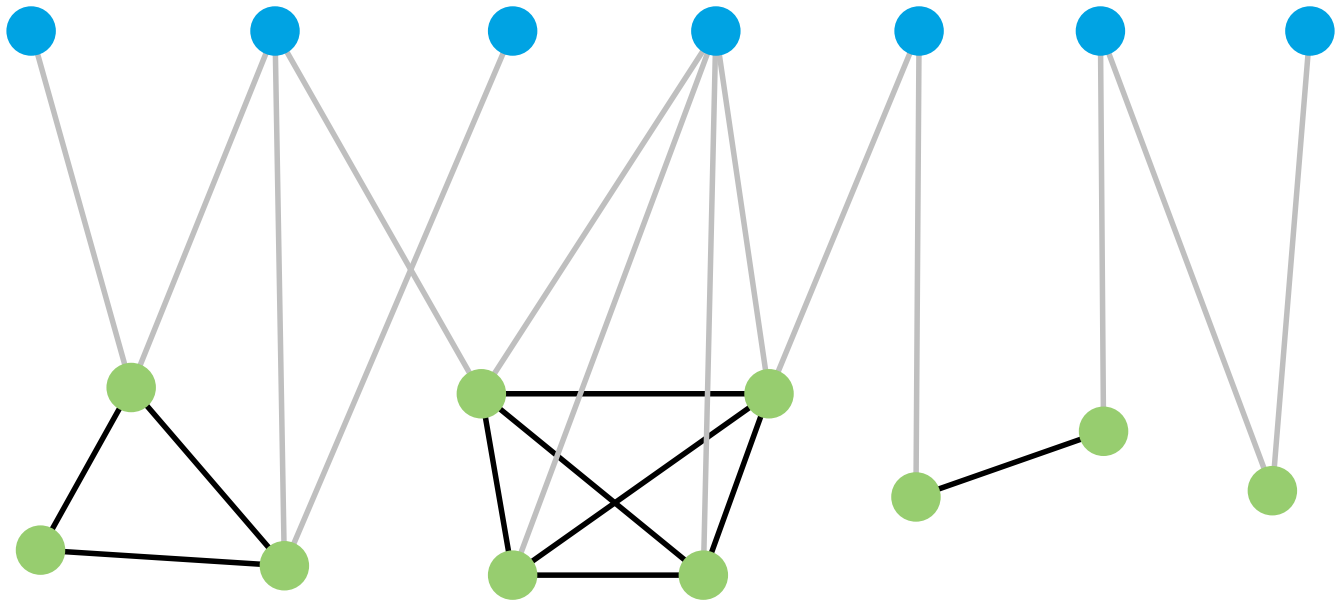
# A monopolar partition



A graph is *monopolar* if its vertices can be partitioned into $(A, B)$ so $A$ is an independent set and $B$ is a disjoint union of cliques.

# A monopolar partition



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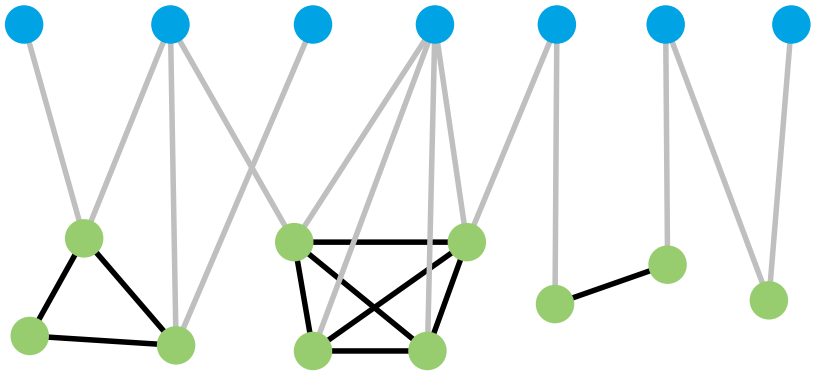
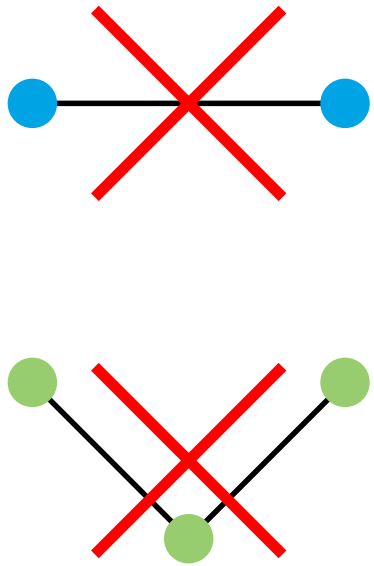


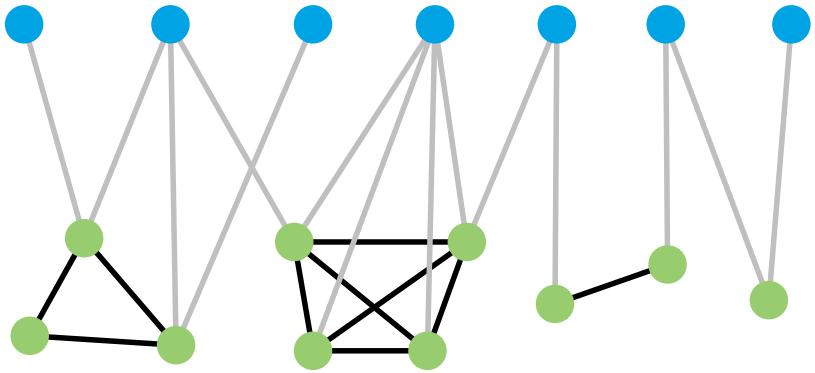


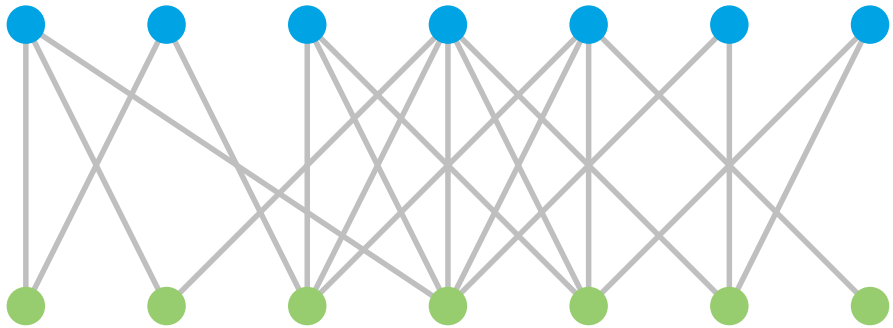
A graph is *monopolar* if its vertices can be partitioned into $(A, B)$ so $A$ contains no induced $P_2$ and $B$ contains no induced $P_3$.

why should I care?

# Why should I care?

Monopolar graphs generalize bipartite graphs





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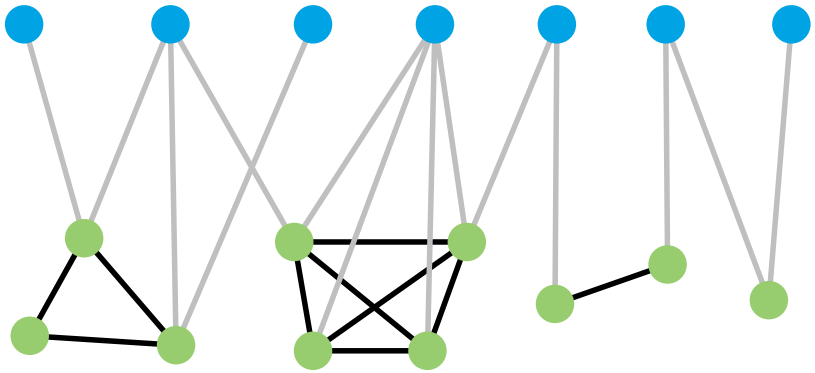
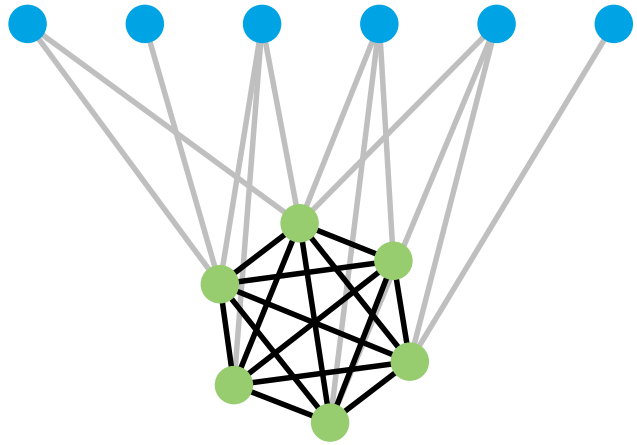
Monopolar graphs generalize bipartite graphs and split graphs





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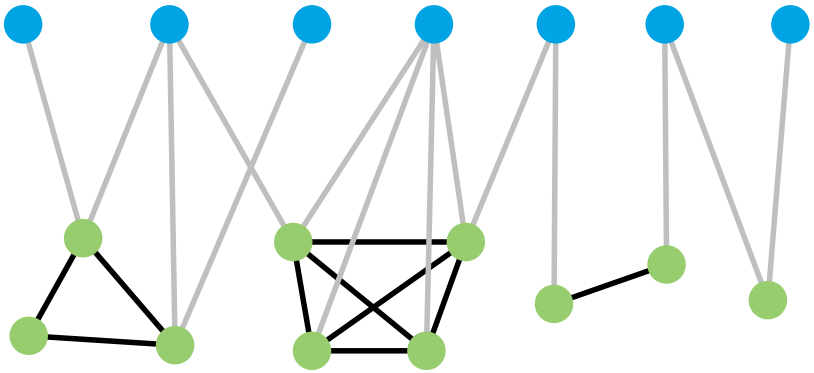
Monopolar graphs generalize bipartite graphs and split graphs and share some of their properties



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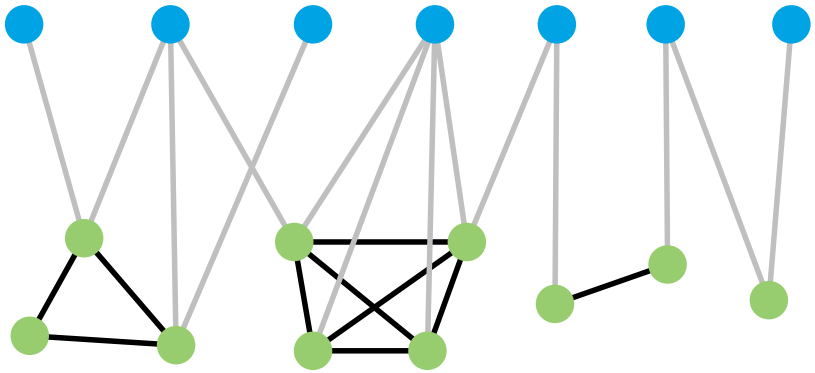

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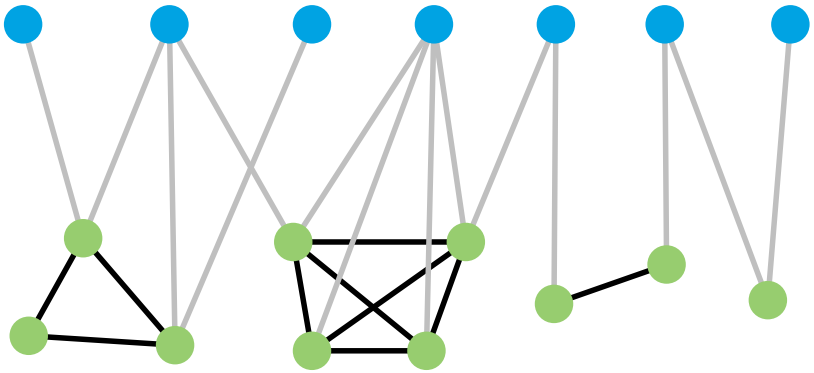
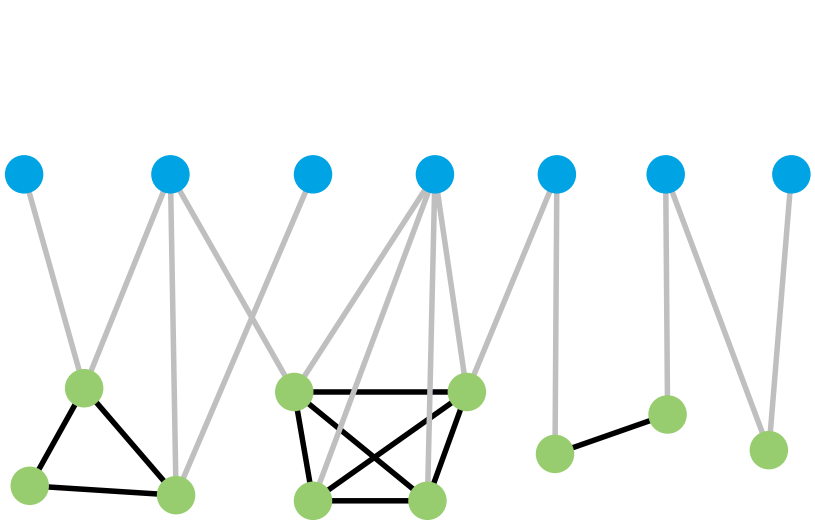
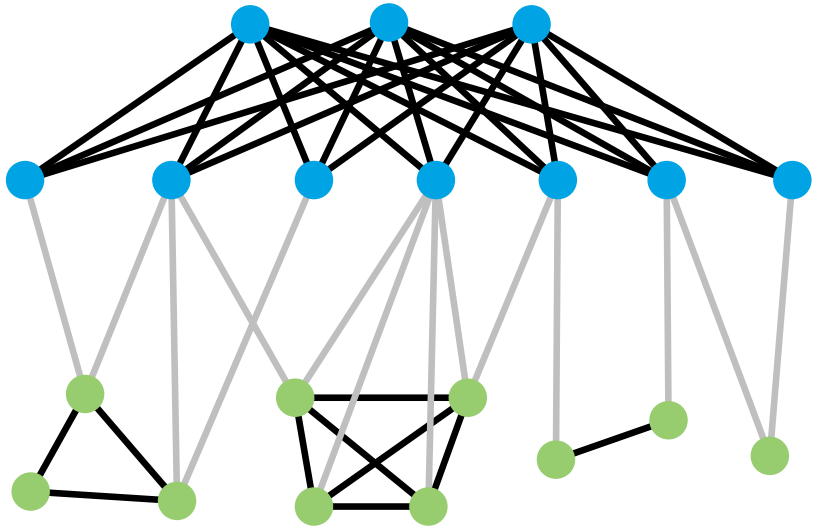


- at most $m$ maximal cliques
- $\chi \leq \omega + 1$
- closed under disjoint union, induced subgraphs

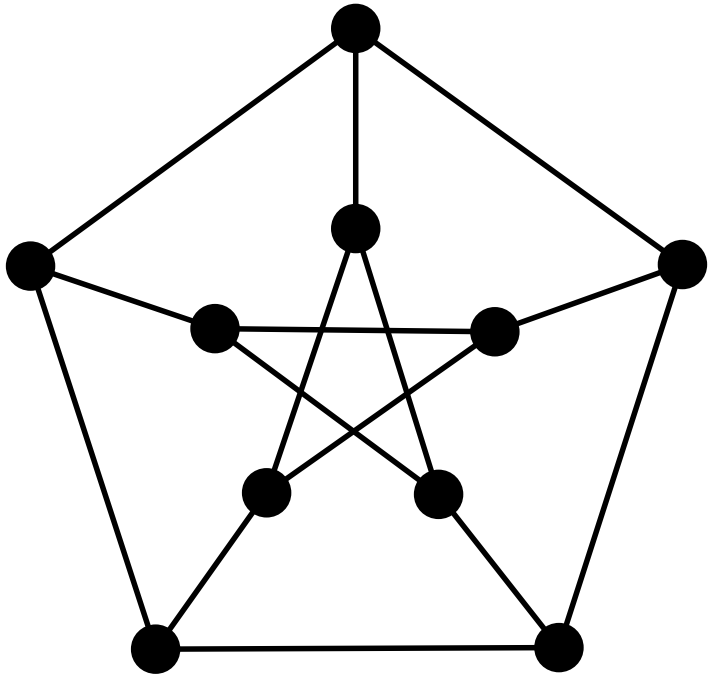
# Why should I care?

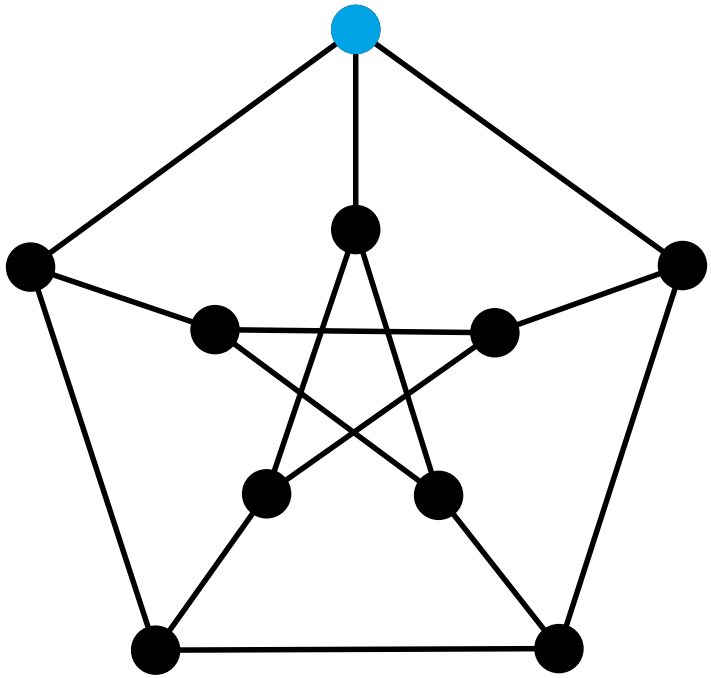
Monopolar graphs are an important subclass of *polar graphs*.

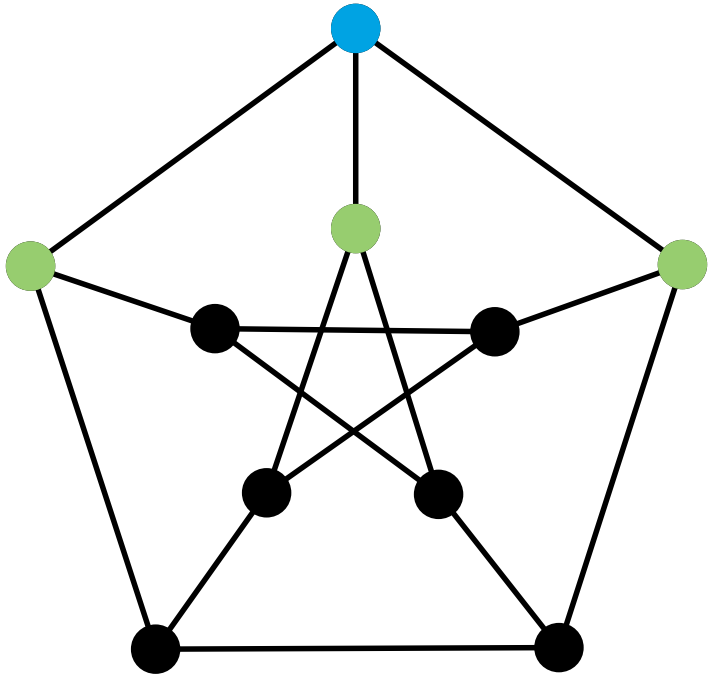


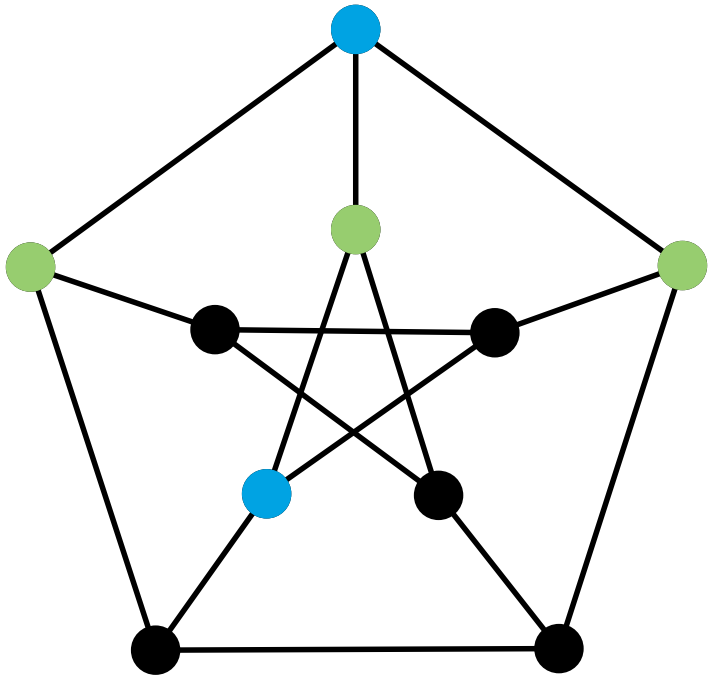


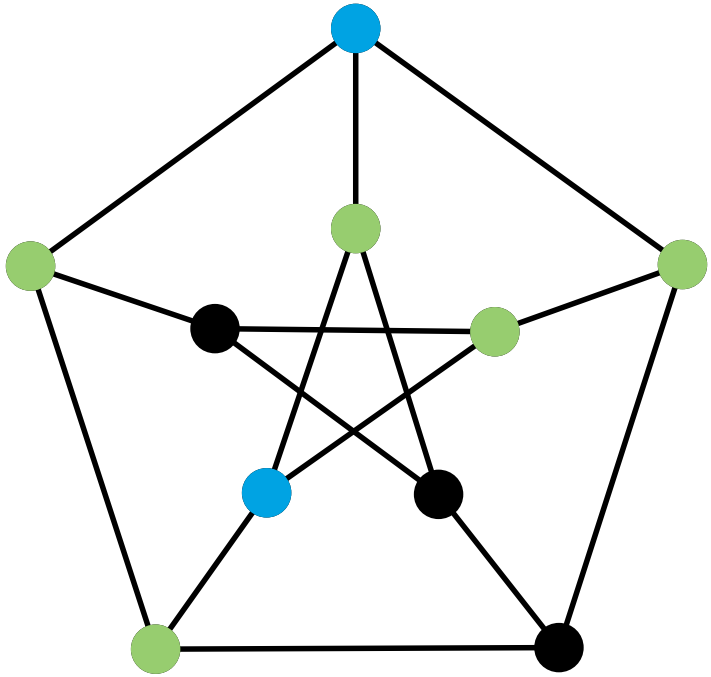
some examples

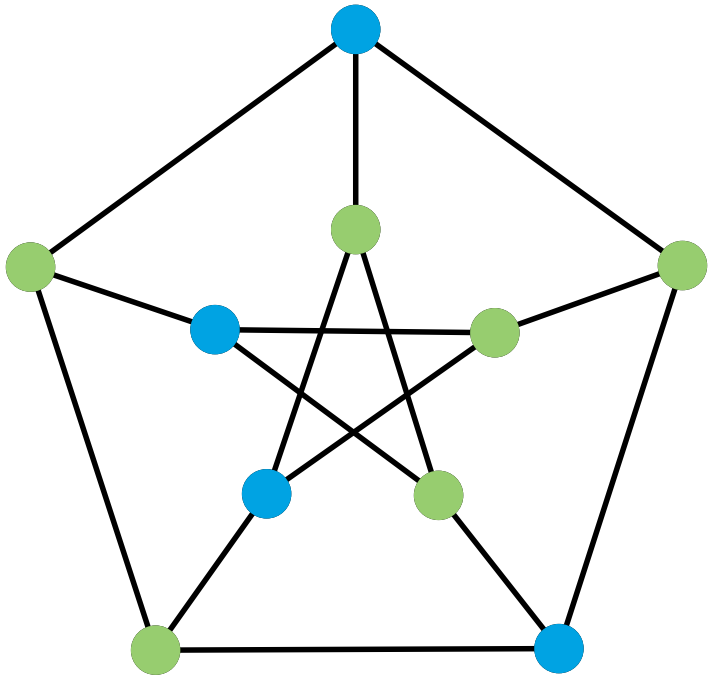


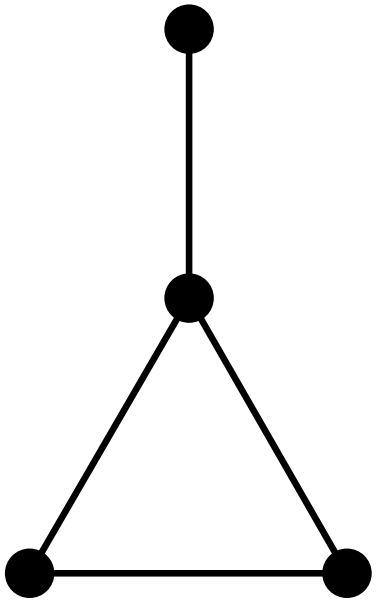


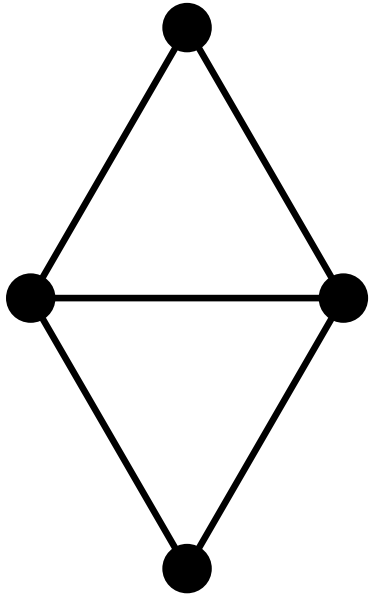


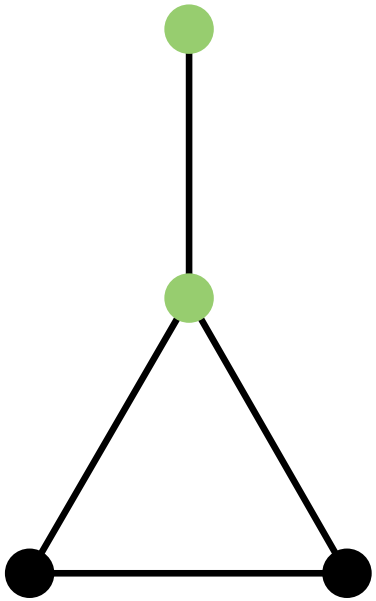


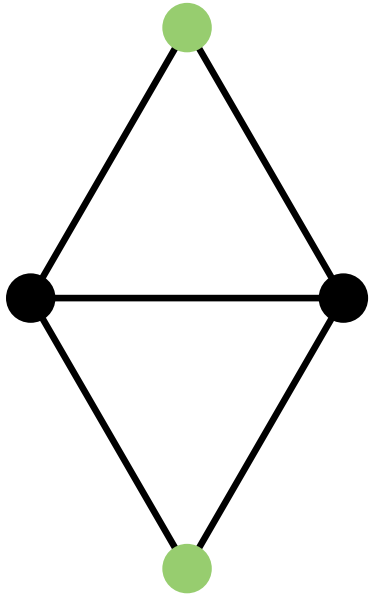


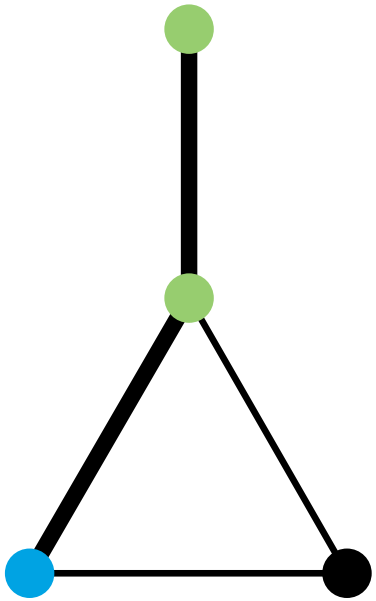


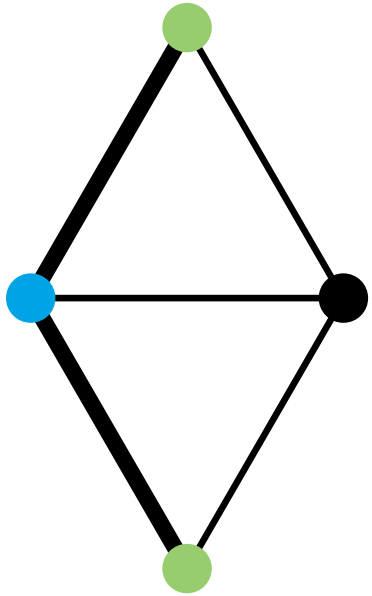


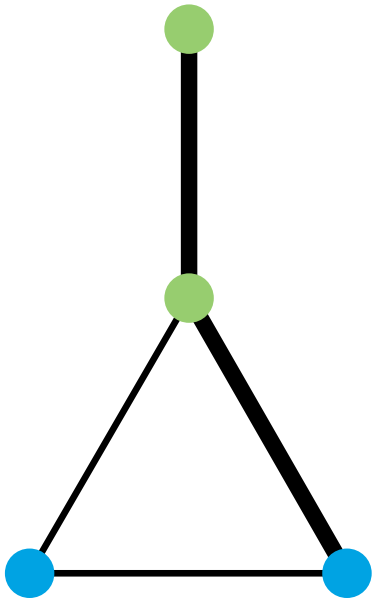


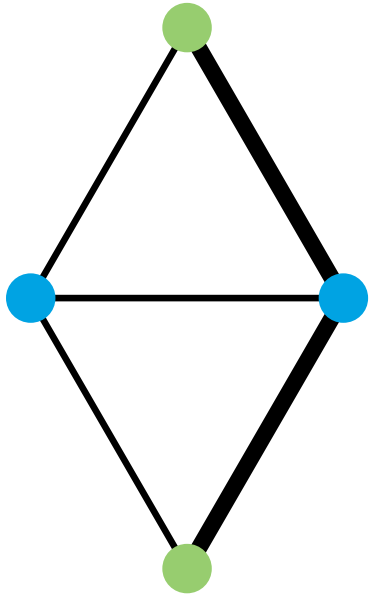


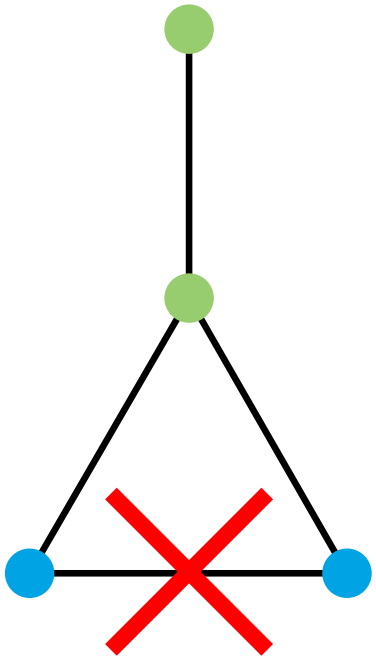


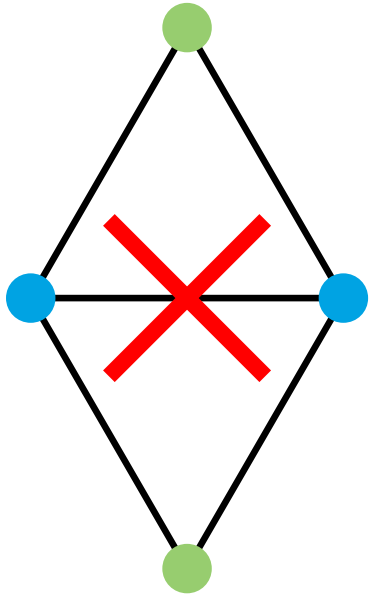


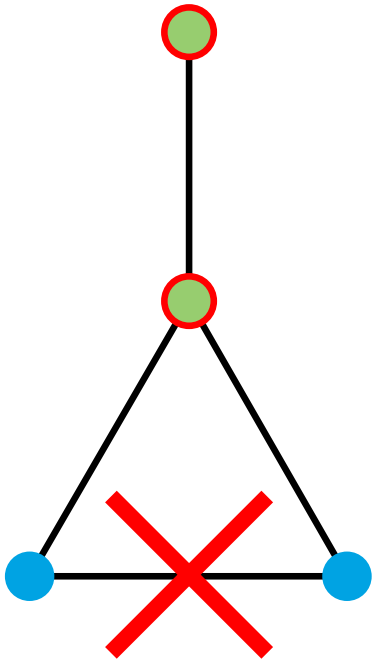


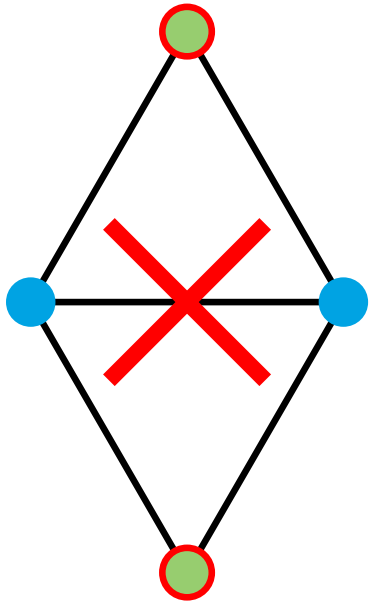


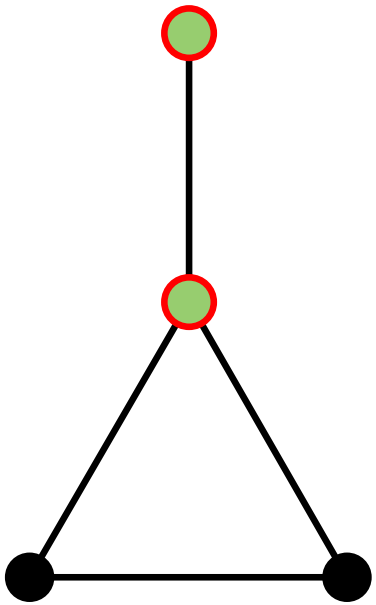


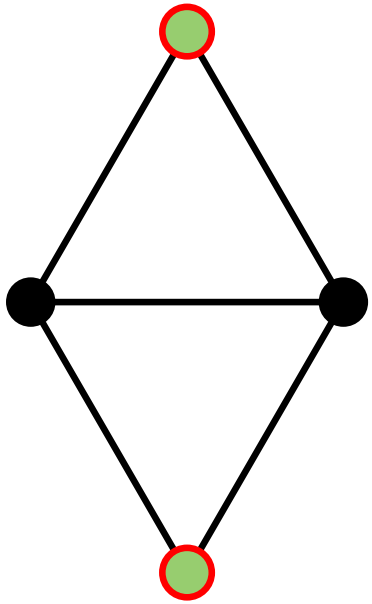




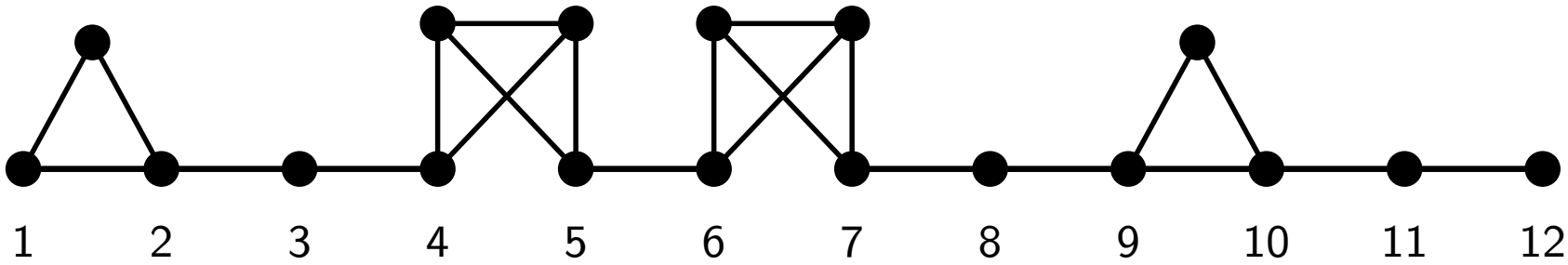








In an induced paw or kite, we have two **specified vertices** which cannot both be in $B$.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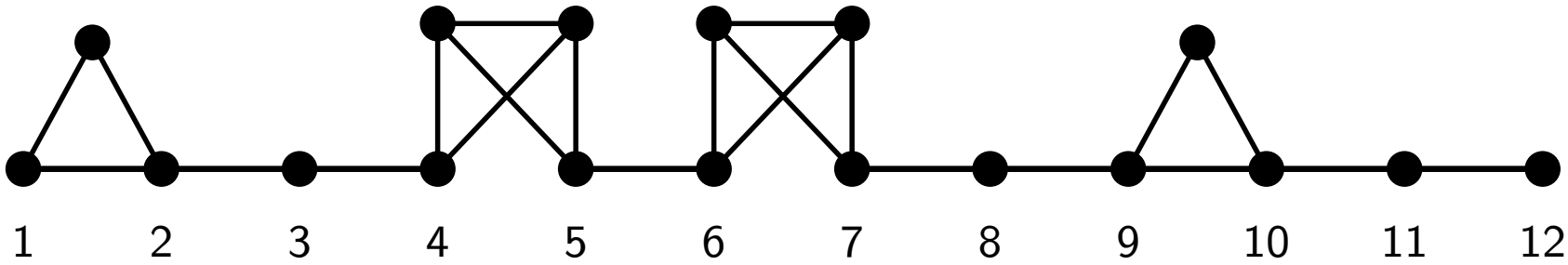
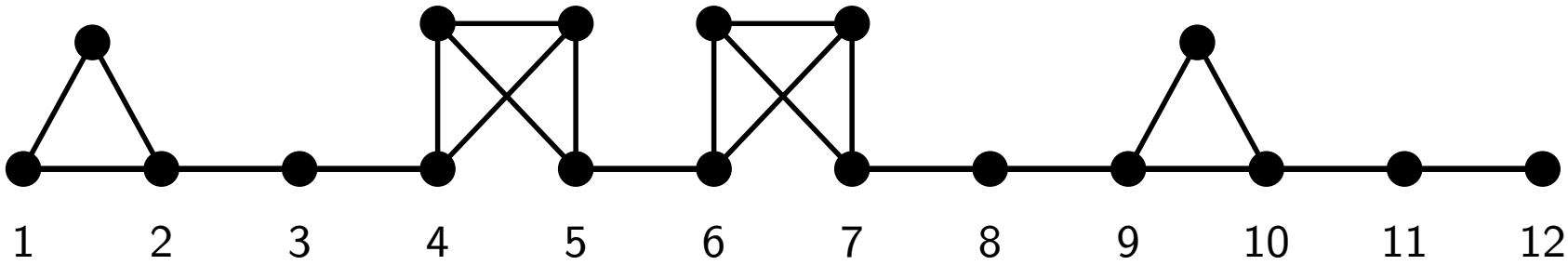



A sequence of vertices $v_1, v_2, \ldots, v_k$ is called **well-formed** if:

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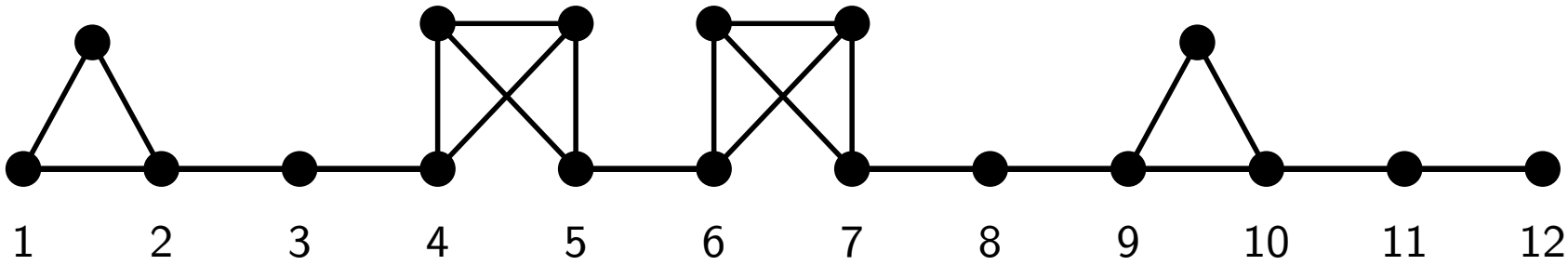
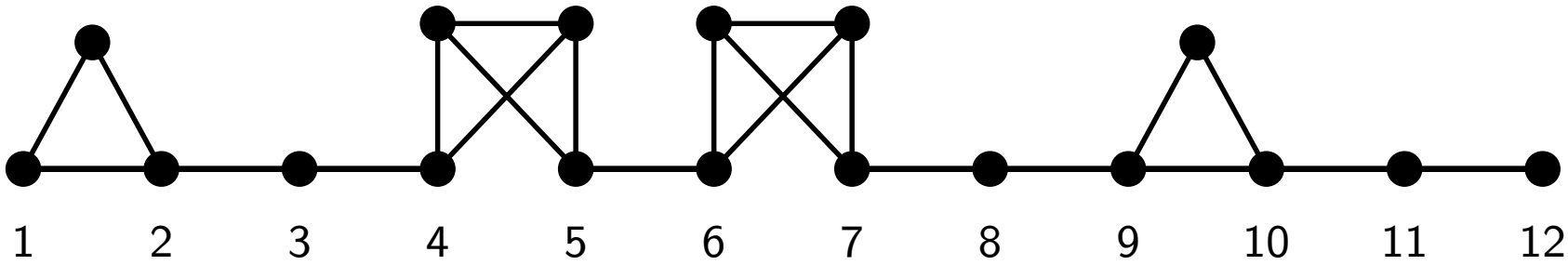
- $v_{i-1}v_i$ is an edge (for each even $i$)




A sequence of vertices $v_1, v_2, \ldots, v_k$ is called **well-formed** if:

- $v_{i-1}v_i$ is an edge (for each even $i$)
- $v_{i-1}, v_i$ are specified vertices of a paw or kite (for each odd $i$)




A sequence of vertices $v_1, v_2, \ldots, v_k$ which is **well-formed** has:

- $v_{i-1}, v_i$ not both in $A$ (for each even $i$)
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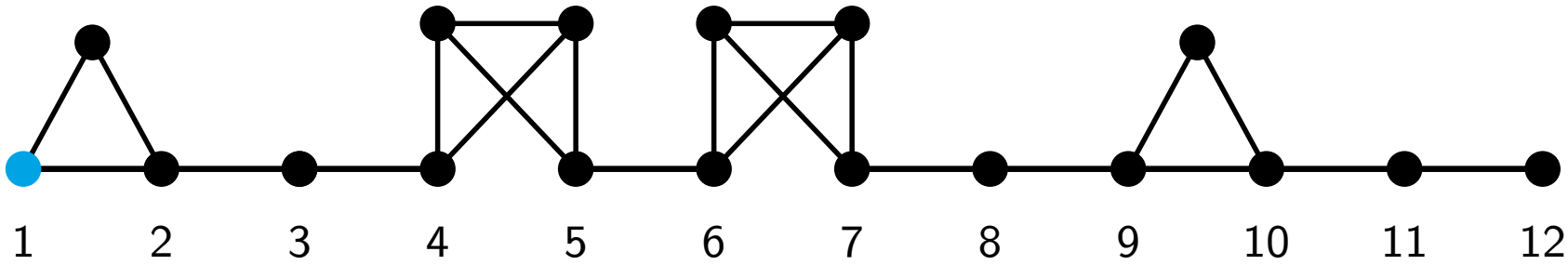


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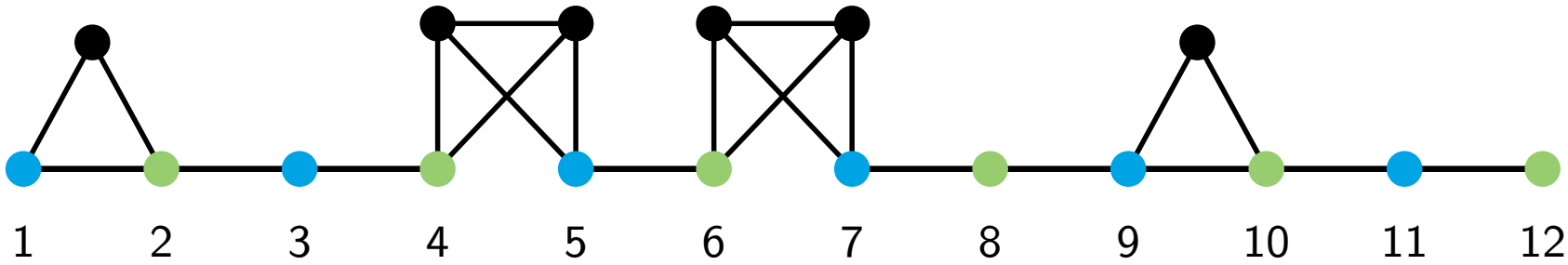


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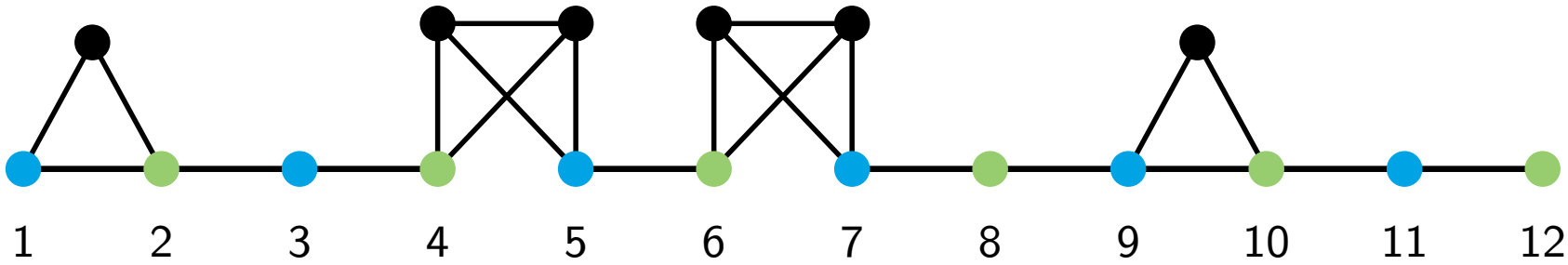


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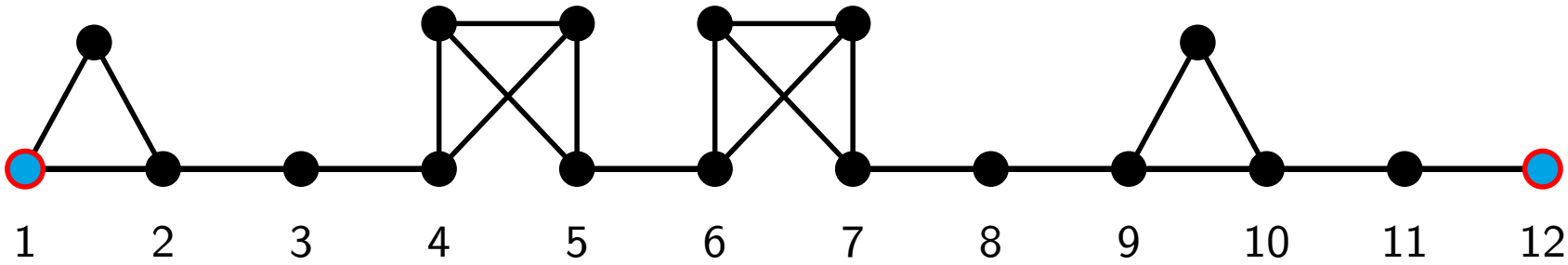
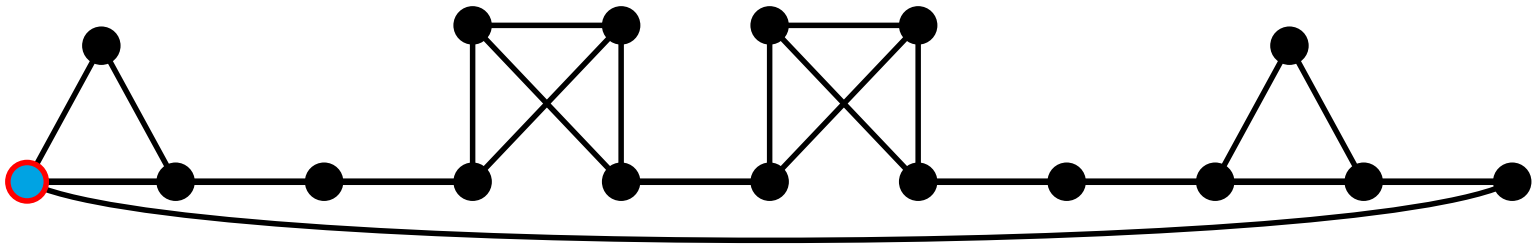
**Proposition**

If the first vertex of a well-formed sequence is in $A$, then all odd-indexed vertices are in $A$ and all even-indexed vertices are in $B$.




An even-indexed vertex is **incompatible** with the first vertex: they cannot both be in $A$.



An even-indexed vertex is **incompatible** with the first vertex: they cannot both be in $A$.

A vertex might be **self-incompatible** if it is repeated in the sequence with odd and even index: such a vertex must be in $B$.

our necessary conditions

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The set of self-incompatible vertices in any monopolar graph induces a disjoint union of cliques.

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# claw-free graphs

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If $(A, B)$ is a monopolar partition, then $A$ is a compatible set: no vertices are joined by a well-formed sequence of even length.

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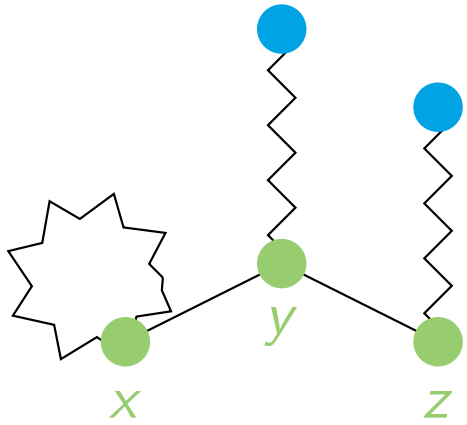

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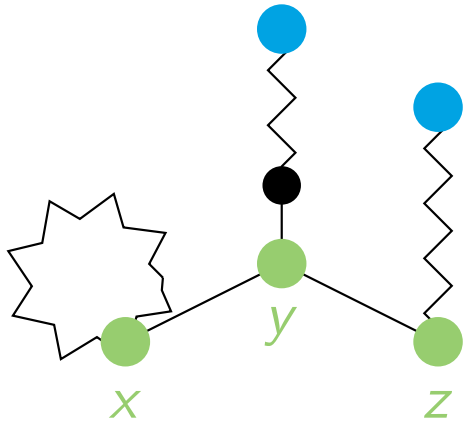

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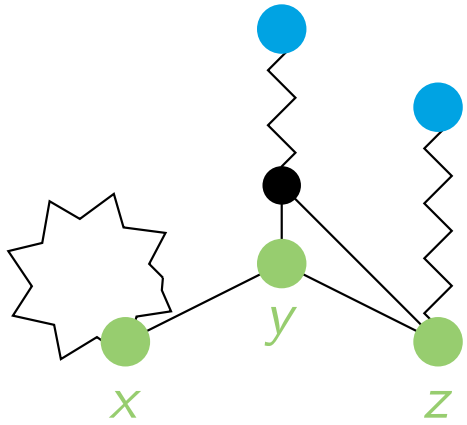

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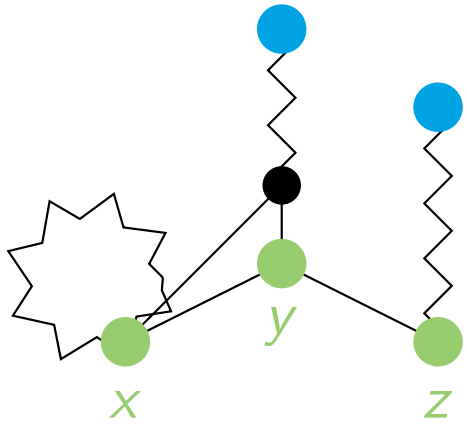

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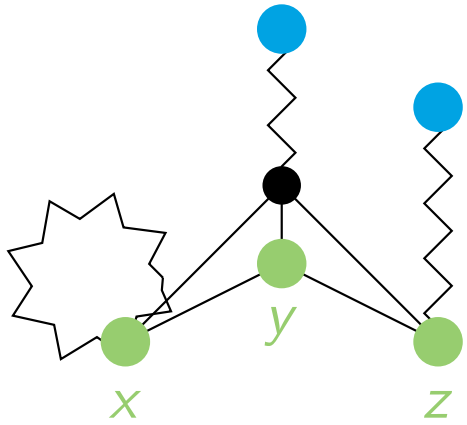
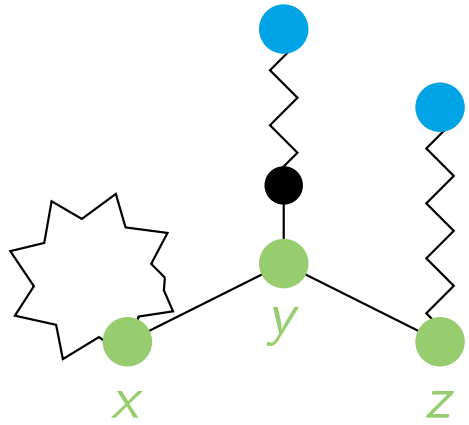


Take $A$ to be a maximal compatible set. If $V(G) \setminus A$ is not a disjoint union of cliques...




...we can find a well-formed sequence between vertices in $A$, a contradiction. So $(A, V(G) \setminus A)$ is a monopolar partition. □

# algorithm

Previous algorithm [Ekim]: $O(n^4 m^2)$

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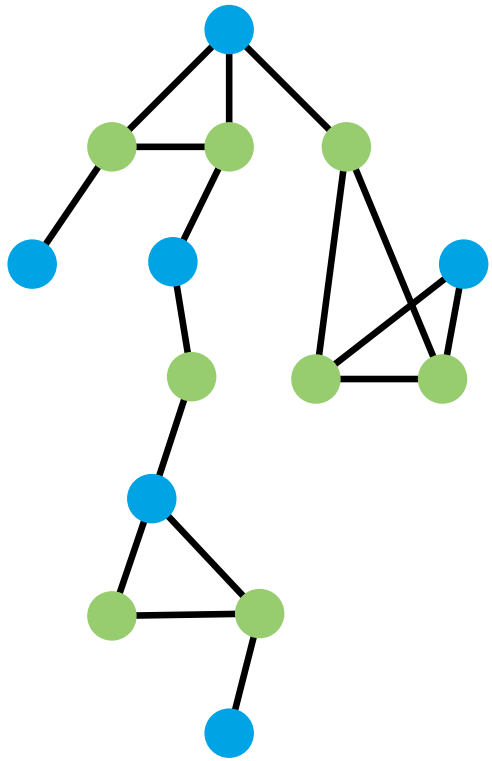

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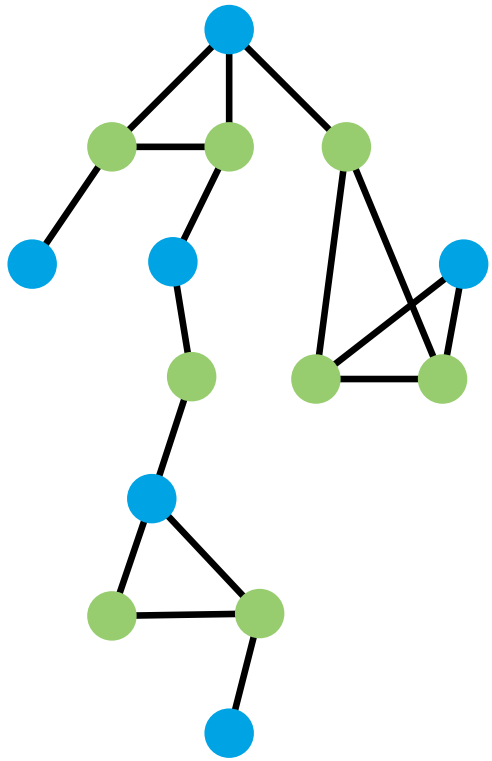
With this characterization: $O(n^3)$

In general: NP-complete



for more on monopolarity,

www.rosschurchley.com

# Monopolar claw-free graphs

Ross Churchley

University of Victoria

December 4, 2010